

# Human Fuzzy Control: Action Dynamical Trap Model

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We consider a general problem related to the mathematical description of human fuzzy rationality. Human operators controlling dynamical systems are often incapable of precisely identifying and implementing the desired control strategy. The operator of a dynamical system treats the current value of the control effort as acceptable if it deviates insignificantly from the desired, or optimal value. The operator starts correcting the actions only when she identifies that this deviation has become considerably large. We propose a non-Newtonian model capturing the operator fuzzy perception of her own actions based on the previously introduced dynamical trap concept. It deals with the physical phase space of a controlled system extended with an independent phase variable characterizing the operator motivated actions. The properties of the model are illustrated via the simple example of an oscillator with action dynamical trap.

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## I. INTRODUCTION

The basic physical notions and mathematical formalism have been successfully employed in modeling social and psychological phenomena. The notions of Newtonian mechanics were used in social force models for traffic dynamics and crowd behavior [1, 2]. The statistical physics framework, namely, the master equation approach, has been widely used in describing the opinion formation and language evolution (see, e.g., review [3]). An extended discussion of applying various techniques inherited from physics to social systems can be found in [4]. Nonetheless, despite the gained success in describing social phenomena in mathematical terms, up to now there is a strong demand for notions and models reflecting the unique properties of human beings [5]. Such features as feelings, emotions, intentions, and beliefs distinguish humans from inanimate objects studied in physics. Development of specific notions and formalism capturing the peculiarities of human behavior at the level of individuals can enable us to model, simulate and better understand complex social phenomena met in everyday life.

One of the cornerstones of modern physics widely met in social psychology [5] is the notion of fixed-point attractor, or equilibrium. For instance, a person achieving and maintaining a certain end-state or goal can be formally treated as a dynamical system drifting towards an equilibrium point in the corresponding phase space [6]. Likewise, one may consider an entity controlled directly by a human operator whose purpose is to maintain its stability. In this case the system dynamics as a whole also can be described by a fixed-point attractor. External or internal factors may cause the system to deviate from the equilibrium, but if the operator is capable of handling such perturbations, the system will eventually

evolve to the desired state.

Recent advances in the field of human control give evidence to the fact that humans do not generally operate the systems under their control in a precise way. Maintaining the system exactly at the desired position requires the ability of the operator to keep perfect awareness and to react immediately even to the smallest deviations. Meanwhile, experimental studies have revealed that the considerable response latency and the effects of noise in the sensorimotor system prevent human operators from implementing continuous control strategies (see, e.g., [7] and references therein). Instead, the discontinuous, or intermittent control is found to be efficient in the presence of time delays and random perturbations in human-controlled processes [8]. The “drift and act” pattern of human control has been detected, e.g., in aircraft landing [9], stick balancing at the fingertip [10] and postural control during quiet standing [11].

In each of these processes human operator prefers to ignore small deviations of the dynamical system from the desired state, starting the active control over the system only when the deviation becomes too large to ignore. The reasons for such behavior vary depending on the properties of the particular system. For instance, while controlling the systems that are relatively sensitive to human response (e.g., balancing a stick), operators ignore small deviations in order not to destabilize the system by the imprecise corrective actions [7]. In the processes with relatively slow dynamics (e.g., car following) human operators tend to “satisfice” rather than to optimize [12]. An operator prefers to stay relaxed while the current situation is acceptable, taking the control over the system only when she is uncomfortable with the deviation from the desired state.

The aforesaid allows us to conclude that human operators, at least in some situations, do not distinguish between the optimal state of a controlled system and sub-optimal states from its vicinity. This property of human behavior is a manifestation of the phenomenon often referred to as human fuzzy rationality [13]. In the general

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case the operator fuzzy rationality makes the standard equilibrium point formalism not applicable for describing the dynamics of human-controlled systems. Instead, such systems exhibit some complex time-dependent patterns of behavior near the virtual equilibrium [14]. Up to now there have been a few attempts to develop a mathematical formalism capturing the effects of human fuzzy rationality. In particular, the basic model of human reaction threshold is commonly used in applied studies [9, 10, 15, 16], but still there is a lot of uncertainty about the intrinsic mechanisms causing the anomalous behavior of the systems under human control (see, e.g., [17]). The dynamical trap model [18–20] being a certain version of the fuzzy threshold concept is another alternative to the standard fixed-point attractor in complex sociopsychological systems. Both of the two models capture the fuzziness of the desired end-state by introducing a certain region (Fig. 1, right frame) around the virtual equilibrium, where each state is treated as acceptable by the operator. However, these models consider the operator behavior to be strictly rational outside the acceptable states region. It means that in driving the system towards this region the operator generally follows some predefined control law. Let us, for example, consider a physical system whose dynamics is specified by its coordinate  $x$  and velocity  $v = dx/dt$ . Then the actions of a strictly rational operator can be represented as a certain function  $a = a(x, v)$  of the phase variables  $\{x, v\}$ , maybe with some time delay. As a result, the behavior of such a system governed by the operator actions is completely described by a differential equation similar to  $\dot{v} = F(x, v, a)$ .

Still, one may claim that humans are often unable to implement the desired control strategy  $a(x, v)$  precisely. Rather, the operator is only able to determine if she is either comfortable with the current value of control parameter, or feels that it is worth changing. In other words, at each instant there is a whole fuzzy set of control parameter values acceptable for the operator (Fig. 2). The common approach to modeling such effects is to introduce an additive noise term (see, e.g., [21]), that is justified in some situations. However, this approach does not reflect some characteristic features of human control (e.g., the on-off intermittency [8]). Besides, a wide class of intricate phenomena observed in the systems of interacting individuals still remains unexplained.

In the present paper we argue that the effect of human fuzzy rationality extends beyond the desired end-state concept to the notion of action strategy. Appealing to the dynamical traps framework we try to capture the fluctuations of the operator actions in the vicinity of the virtual optimal control strategy. The two-dimensional system phase space  $(x, v)$  is extended with acceleration  $a$  as an independent phase variable that is perceived, evaluated, and indirectly controlled by the operator. By cost of introducing the non-Newtonian variable  $a$  we gain the possibility of describing mathematically the fuzzy set of acceptable suboptimal action strategies which is further

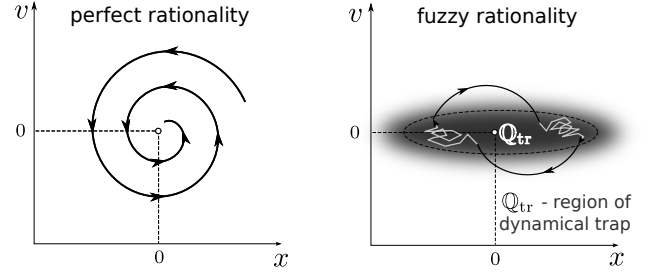


FIG. 1: The phase space structure of the dynamical system under human control in case of perfect and fuzzy operator rationality.

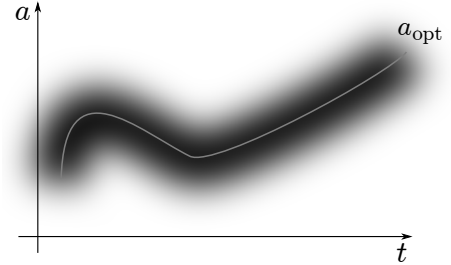


FIG. 2: Action dynamical trap: the vicinity of the optimal strategy  $a_{\text{opt}}(t)$  in the space of all action strategies  $\{a(t)\}$ .

referred to as the action dynamical trap.

### Model background

Hereafter we explain the basic idea of the dynamical trap model by employing a simple example of a noisy human-controlled dynamical system with no internal dynamics. The phase space of the system comprises the coordinate  $x$  and the velocity  $v$ . The goal of the operator is to maintain the system at the desired state, the origin, by implementing the optimal in some sense control strategy  $a_{\text{opt}}(x, v)$ . However, if the system currently resides in some vicinity of the desired state, the operator prefers to halt active control over the system. The equations describing the system dynamics under the operator control are written as follows

$$\begin{aligned} \dot{x} &= v, \\ \dot{v} &= \Omega(x, v)a_{\text{opt}}(x, v) + \varepsilon f(t), \end{aligned} \quad (1)$$

where  $f(t)$  is the random force with the amplitude  $\varepsilon \ll 1$ . The cofactor  $\Omega(x, v)$  is some function such that  $\Omega(x, v) \approx 1$  for all the values  $(x, v)$  that are far enough from the origin and  $\Omega(x, v) \ll 1$  in a certain neighborhood  $Q_{\text{tr}}$  of the origin (Fig. 1, right frame).

In order to explain the meaning of the cofactor  $\Omega(x, v)$  we consider the behavior of the operator approaching the desired phase space position  $(x = 0, v = 0)$ . When the current position is far from the origin, the operator perfectly follows certain optimal action strategy  $a_{\text{opt}}(x, v)$ .

If the current position is recognized by the operator as “good enough”, i.e.,  $(x, y) \in \mathbb{Q}_{\text{tr}}$ , she halts active control over the system. So, during a considerable period of time the system is affected only by random factors of a small amplitude; in other words, the system is “trapped” in a vicinity of the desired position. Therefore,  $\mathbb{Q}_{\text{tr}}$  is called the area of dynamical trap. One may notice that in the case of negative linear feedback control strategy ( $a_{\text{opt}} \propto x + \sigma v$ , where  $\sigma$  is a constant) the given system under human control is analogous to the physical system of a damped harmonic oscillator. This allows us to call system (1) the oscillator with dynamical trap.

The oscillator with dynamical trap captures the basic behavior properties of the fuzzy rational operator, i.e., the operator who does not react to small deviations from the desired position in the phase space. When the deviation becomes large enough, the operator decides to start controlling the system in order to return it to an acceptable state. This can be achieved by varying the control parameter, namely, the acceleration, in a way that is optimal in some sense.

Let us appeal to the car following, which is a characteristic example of a dynamical system governed by operator with fuzzy rationality [22]. Car drivers are unable to continuously keep perfect awareness of the surrounding situation, so they usually set the acceleration to some constant value based on the current circumstances. Once fixed, the value of acceleration is changed only when the driver realizes that the deviation from some “optimal” acceleration value has become too large to be ignored. In other words, considerable deviations of the current acceleration  $a$  from the optimal value  $a_{\text{opt}}$  cause the operator to start active control over the car motion. However, when the difference  $a - a_{\text{opt}}$  is rather small, there are no stimuli for the driver to act, i.e., to change the acceleration. Thus, one may imagine a certain region around the optimal strategy  $a_{\text{opt}}(x, v)$ , wherein each strategy is regarded as acceptable (Fig. 2). Instead of precisely following the optimal strategy, the operator just keeps the actually implemented strategy inside this region, making some corrections only when the mismatch  $a - a_{\text{opt}}$  exceeds some fuzzy threshold. For this reason the region of acceptable strategies around  $a_{\text{opt}}$  will be called the action dynamical trap. The “thickness” of the action dynamical trap is determined by the capacity of the operator perception and her levels of concentration and motivation to follow the optimal control strategy. The action dynamical trap model is proposed to capture the discussed effects of fuzzy rationality in choosing and implementing the action strategies in human-controlled dynamical processes.

## II. ACTION DYNAMICAL TRAP

We start our speculations from considering the original dynamical trap model described by equations (1), where functions  $f(t)$ ,  $a_{\text{opt}}(x, v)$  and  $\Omega(x, v)$  are to be specified.

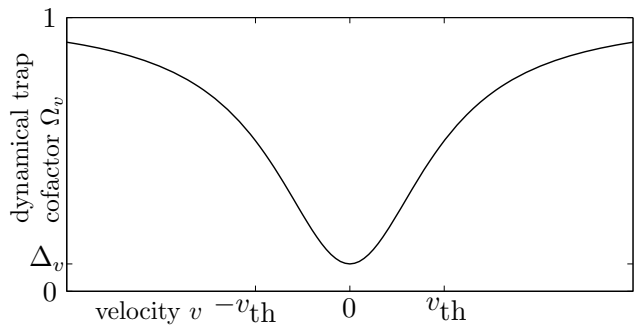


FIG. 3: The dynamical trap cofactor  $\Omega_v(v)$ .

First of all, in order to elucidate the basic properties of the model, we exclude the random factors from the scope of the present paper, i.e., we consider  $f(t) \equiv 0$ . Then, following [20], we define the operator control strategy as a linear feedback aimed at maintaining the system at the origin:  $a_{\text{opt}}(x, v) = \omega^2(x + \frac{\sigma}{\omega}v)$ , where  $\omega$  and  $\sigma$  are constant coefficients. Finally, the dynamical trap cofactor  $\Omega(x, v)$  is claimed not to depend on  $x$ . It reflects the assumption that the control over the system velocity  $v$  is of prior importance for the operator comparing to the control over the coordinate  $x$ . We use the ansatz

$$\Omega(x, v) := \Omega_v(v) = \frac{\Delta_v v_{\text{th}}^2 + v^2}{v_{\text{th}}^2 + v^2}, \quad (2)$$

where  $\Delta_v$  is the dynamical trap intensity coefficient and  $v_{\text{th}}$  is the threshold value of velocity (Fig. 3).

The pivot point of the proposed approach is that we regard human actions as an independent component of the system rather than some predetermined function of its physical state. We extend the physical phase space  $\{x, v\}$  by introducing a new phase variable, in the given case, the system acceleration  $a$ , i.e.,

$$\{x, v\} \rightarrow \{x, v, a\}.$$

It enables us to ascribe to the system an additional degree of freedom corresponding to the operator actions.

The model capturing the dynamical trap effect in controlling the deviation  $a - a_{\text{opt}}$  is written as

$$\begin{aligned} \dot{x} &= v, \\ \dot{v} &= a, \\ \tau \dot{a} &= -\Omega_a(a - a_{\text{opt}}(x, v))(a - a_{\text{opt}}(x, v)). \end{aligned} \quad (3)$$

One may notice that if we set  $\Omega_a(a - a_{\text{opt}}(x, v)) \equiv 1$ , system (3) describes following the optimal action strategy  $a_{\text{opt}}$  precisely by the operator whose reaction time is  $\tau$ . As the human operator is not capable of doing it in the case of small deviations  $a - a_{\text{opt}}$ , we write

$$\Omega_a(a - a_{\text{opt}}) = \frac{\Delta_a a_{\text{th}}^2 + (a_{\text{opt}} - a)^2}{a_{\text{th}}^2 + (a_{\text{opt}} - a)^2},$$

where, in analogy to (2),  $\Delta_a$  is the dynamical trap intensity and  $a_{\text{th}}$  is the threshold in perceiving acceleration

deviations from the optimal value. For the sake of simplicity, we have redefined the optimal control strategy in such way that it incorporates the dynamical trap effect in correcting the velocity variations:

$$a_{\text{opt}}(x, v) = -\Omega_v(v)\omega^2(x + \frac{\sigma}{\omega}v),$$

where  $\Omega_v(v)$  is specified by expression (2). Thus, the control strategy  $a_{\text{opt}}$  is optimal from the standpoint of fuzzy rational human operator.

Finally, in order to reduce the number of system parameters we change the time and spatial scales as follows

$$t \rightarrow t \frac{1}{\omega}, \quad x \rightarrow x \frac{a_{\text{th}}}{\omega^2}.$$

It is easy to check that in these dimensionless units parameters  $\omega$  and  $a_{\text{th}}$  are both equal to unity. Thus, the above expressions for  $\Omega_a$  and  $a_{\text{opt}}$  take form

$$\begin{aligned} \Omega_a(a - a_{\text{opt}}) &= \frac{\Delta_a + (a_{\text{opt}} - a)^2}{1 + (a_{\text{opt}} - a)^2}, \\ a_{\text{opt}}(x, v) &= -\Omega_v(v)(x + \sigma v). \end{aligned} \quad (4)$$

System (2)–(4) possesses the only equilibrium point at the origin. The standard linear stability analysis reveals that this equilibrium is stable for all the values of the system parameters  $\sigma$ ,  $\tau$ , and  $\Delta_a$  such that

$$\frac{\tau}{\sigma} > \Delta_a. \quad (5)$$

If the effect of the action dynamical trap is weak,  $\Delta_a \approx 1$ , the system is stable for  $\tau < \sigma$ , i.e., when the operator reaction time  $\tau$  is relatively small and the capability of suppressing the velocity deviations  $\sigma$  is relatively high.

In the case of strong effect of the action dynamical trap,  $\Delta_a \ll 1$ , system (2)–(4) is stable only if  $\tau \ll \sigma$ . This may be interpreted in such sense that the operator can not precisely maintain the desired state of the system, unless her reaction is almost immediate ( $\tau \ll 1$ ) or the velocity feedback gain  $\sigma$  is extremely large. Therefore, normally the system governed by equations (2)–(4) is unstable at the origin for high intensity of the action dynamical trap. It is notable that the system stability does not depend on the parameter  $\Delta_v$  quantifying the intensity of the velocity dynamical trap.

### III. NUMERICAL RESULTS

In the present paper we focus our attention on the case of strong effect of both the velocity and acceleration dynamical traps ( $\Delta_v = \Delta_a = 0.001$ ); the dependence of the system behavior on the dynamical trap intensity is the subject of individual analysis. Also, as mentioned previously, we refrain from investigating the influence of the stochastic factors on the system dynamics, as this is worthy of individual study as well.

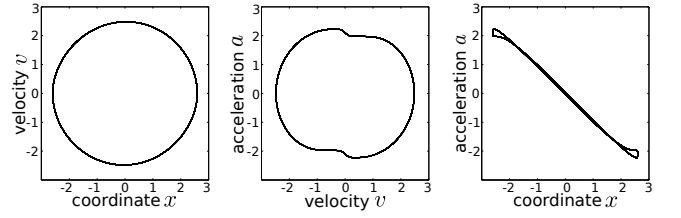


FIG. 4: The projections of the limit cycle formed by the phase trajectory of system (2)–(4). The values of parameters used for simulation are  $\sigma = 1$ ,  $\tau = 0.9$ ,  $v_{\text{th}} = 0.2$ .

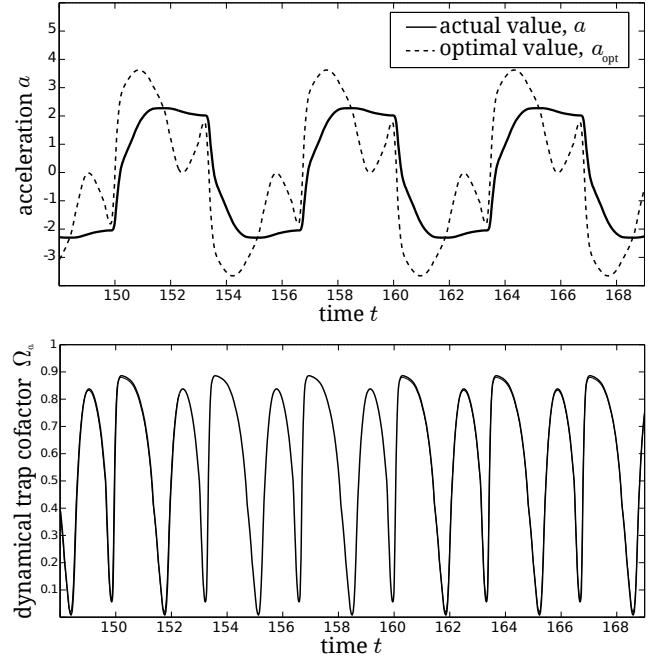


FIG. 5: Time pattern fragments of the actual and optimal acceleration (top frame) and corresponding dynamical trap cofactor (bottom frame). The fragments match the phase portrait shown in Fig. 4.

We analyzed the behavior of system (2)–(4) numerically under the adopted assumptions for various values of system parameters. The absolute and relative error tolerance parameters of the routine used for the numerical simulations were chosen in such way that varying them tenfold could not affect the results of the simulations. The initial conditions for simulations were formed by assigning random values to the system coordinate and velocity. We observed two major patterns of system dynamics depending on the parameters  $\sigma$ ,  $\tau$  and  $v_{\text{th}}$ .

First, for all the checked values of parameters such that  $\Delta_a \leq \tau/\sigma < 1$  the system exhibits periodic oscillations after a short transition process. The found limit cycle seems to be the only stable periodic attractor for all the tested values of the parameter  $v_{\text{th}}$ ; it was checked by varying the amplitude of the initial random perturbations by  $10^6$  times. The example of such cycles found for  $\sigma = 1$ ,  $\tau = 0.9$ ,  $v_{\text{th}} = 0.2$  is represented in Fig. 4. The fragment

of the acceleration time pattern  $a(t)$  corresponding to this phase portrait is depicted in the top frame of Fig. 5, as well as is the evolution of the optimal action strategy  $a_{\text{opt}}(t)$ . As clearly seen, the implemented action strategy remains in the vicinity of the optimal one. When the difference between these two strategies becomes sufficiently small, the time increment of the acceleration growth is also small. It reflects the fact that under this condition the operator almost does not change the control variable,  $a$ , for a certain period of time. However, when the deviation from the optimal action strategy becomes large, the operator behavior turns to be active and the actual acceleration changes fast. This is also reflected in the bottom frame of Fig. 5, where the time pattern of the dynamical trap cofactor  $\Omega_a$  is represented. The values of  $\Omega_a$  near unity correspond to the periods of the acceleration active growth/decrease, while the stagnation of  $a$  is characterized by values of  $\Omega_a$  close to zero.

Second, as the ratio  $\tau/\sigma$  exceeds unity, the structure of system phase space becomes more complex and essentially depends on the parameter  $v_{\text{th}}$ . Given that  $v_{\text{th}}$  is below some critical value  $v_{\text{th}} < \hat{v}_{\text{th}}(\tau, \sigma)$ , the system has no attractors. It means that under such conditions the operator just can not control the system dynamics in principle. For  $v_{\text{th}} > \hat{v}_{\text{th}}(\tau, \sigma)$  two limit cycles appear. The first periodic orbit is stable and has the same form as described above. The second periodic orbit appears to be unstable and is located outside the first one. We failed to derive analytically the explicit conditions of the limit cycle non-existence. However, the values of  $\hat{v}_{\text{th}}$  obtained numerically are paradoxically much higher than the velocity variations actually observed during the corresponding simulations. It prompts us to regard the case of  $v_{\text{th}} > \hat{v}_{\text{th}}(\tau, \sigma)$  having no physical meaning.

Summarizing the obtained results, we state that the given system is:

- stable at the origin and has no other attractors in the case of  $\tau/\sigma < \Delta_a$ ;
- unstable at the origin and has a stable limit cycle when  $\Delta_a \leq \tau/\sigma \leq 1$ ;
- unstable at the origin and has no attractors for all other reasonable values of the parameters.

#### IV. CONCLUSION

We propose a new model of human fuzzy rationality. The model is based on the concept of dynamical traps

elaborated previously [18–20] which matches the modern paradigm of discontinuous human control [8] and appeals to the existence of a certain region of acceptable states near the desired phase space position. The present paper argues that the dynamical trap concept is a more general notion and can be extended to the operator perception of her own actions.

A human operator controlling a dynamical system is usually not capable of selecting or calculating the optimal action strategy that would allow her to reach and maintain the desired end-state or goal. However, during the control process the operator is generally able to realize that the currently implemented strategy deviates from the optimal one if this deviation becomes large enough. Once being aware of the mismatch, the operator can adjust her actions until she feels that the current value of control parameter is acceptable. In order to capture this feature of human cognition, we, first, extend the phase space of the dynamical system under human control by considering the control parameter as an independent phase variable. Second, we introduce a certain region alongside the optimal strategy in the space of all action strategies; each strategy within this region is treated as acceptable by the operator. The latter region is called the action dynamical trap.

We study an example that describes the behavior of a human operator trying to suppress oscillations of a simple dynamical system. The results of the theoretical and numerical analysis of the developed model correspond well to the basic properties of human-controlled systems. Particularly, we elucidate the fact that it is mainly the operator reaction time and her capability of suppressing the velocity deviations that determine the system behavior. The system can be precisely stabilized only by operators with exceptional abilities; otherwise, the system exhibits periodic oscillations around the equilibrium point or even may be completely destabilized by the actions of operator. In the previous studies on the dynamical trap effect various complex cooperative phenomena were observed in the ensembles of interacting oscillators. Despite this, no instability has been found yet in the dynamics of such systems describing a single oscillator without noise effects. The presented model allows for this possibility.

The model still requires extensive further development, that is, e.g., considering probabilistic description of the dynamical trap cofactors  $\Omega_{a,v}$  and taking the stochastic factors into account. Nevertheless, we believe the proposed approach is rather general and may be employed in a wide class of models describing the systems in which human actions are of primary importance.

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